

On Some Classes Of Nano $(1,2)^*$ - \tilde{g} -Closed Sets In Nano Bitopological Spaces

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Abstract. In this paper, we introduce a new class of sets namely nano $(1,2)^*$ - \tilde{g} -closed sets in nano bitopological spaces. This class lies between the class of nano $(1,2)^*$ - \tilde{g}_α -closed sets and the class of nano $(1,2)^*$ - α g-closed sets.

Key words and phrases. Nano $(1,2)^*$ - \tilde{g} -closed, Nano $(1,2)^*$ - \tilde{g} -closed, nano $(1,2)^*$ - \tilde{g}_α -closed, nano $(1,2)^*$ -gsp-closed.

1. INTRODUCTION

Levine [7] introduced generalized closed sets in general topology as a generalization of closed sets. Kelly [5] introduced the concepts of bitopological spaces. K. Bhuvaneswari et al [3] introduce on nano generalized pre closed sets and nano pre generalized closed sets in nano topological spaces. A. Pandi [8] introduced some modifications of nano bitopological spaces.

In this paper, we introduce a new class of sets namely nano $(1,2)^*$ - \tilde{g} -closed sets in nano bitopological spaces. This class lies between the class of nano $(1,2)^*$ - \tilde{g}_α -closed sets and the class of nano $(1,2)^*$ - α g-closed sets.

2. PRELIMINARIES

Definition 2.1 [9]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x .

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [6] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 [6] If $[\tau_R(X)]$ is the nano topology on U with respect to X , then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Throughout this thesis, $(U, \tau_{R_{1,2}}(X))$ (briefly, U) will denote nano bitopological space.

Definition 2.5 [8]

Let S be a subset of U . Then S is said to be $\tau_{R_{1,2}}(X)$ -open if $S = A \cup B$ where $A \in \tau_{R_1}(X_1)$ and $B \in \tau_{R_2}(X_2)$.

The complement of $\tau_{R_{1,2}}(X)$ -open set is called $\tau_{R_{1,2}}(X)$ -closed.

Definition 2.6 [8]

Let S be a subset of a nano bitopological space U . Then

- (i) the nano $\tau_{R_{1,2}}(X)$ -closure of S , denoted by $N\tau_{R_{1,2}}(X)\text{-cl}(S)$, is defined as $\bigcap \{F : S \subseteq F \text{ and } F \text{ is } \tau_{R_{1,2}}(X)\text{-closed}\}$.
- (ii) the nano $\tau_{R_{1,2}}(X)$ -interior of S , denoted by $N\tau_{R_{1,2}}(X)\text{-int}(S)$, is defined as $\bigcup \{F : F \subseteq S \text{ and } F \text{ is } \tau_{R_{1,2}}(X)\text{-open}\}$.

Definition 2.7

A subset A of a nano bitopological space U is called

- (i) nano $(1,2)^*$ -semi-open set [4] if $A \subseteq N\tau_{R_{1,2}}\text{-cl}(N\tau_{R_{1,2}}\text{-int}(A))$;
- (ii) nano $(1,2)^*$ - α -open set [2] if $A \subseteq N\tau_{R_{1,2}}\text{-int}(N\tau_{R_{1,2}}\text{-cl}(N\tau_{R_{1,2}}\text{-int}(A)))$;

The complements of the above mentioned open sets are called their respective closed sets.

The nano $(1,2)^*$ -semi-closure (resp. Nano $(1,2)^*$ - α -closure) of a subset A of U , denoted by $N(1,2)^*\text{-scl}(A)$ (resp. $N(1,2)^*\text{-}\alpha\text{cl}(A)$) is defined to be the intersection of all nano $(1,2)^*$ -semi-closed (resp. Nano $(1,2)^*$ - α -closed) sets of U containing A . It is known that $N(1,2)^*\text{-scl}(A)$ (resp. $N(1,2)^*\text{-}\alpha\text{cl}(A)$) is a nano $(1,2)^*$ -semi-closed (resp. Nano $(1,2)^*$ - α -closed) set.

Definition 2.8

A subset A of a nano bitopological space U is called

- (i) nano $(1,2)^*$ -g-closed set [1] if $N\tau_{R_{1,2}}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{R_{1,2}}(X)$ -open in U . The complement of nano $(1,2)^*$ -g-closed set is called nano $(1,2)^*$ -g-open set;
- (ii) nano $(1,2)^*$ -sg-closed set [4] if $N(1,2)^*\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano $(1,2)^*$ -semi-open in U . The complement of nano $(1,2)^*$ -sg-closed set is called nano $(1,2)^*$ -sg-open set;
- (iii) nano $(1,2)^*$ -gs-closed set [8] if $N(1,2)^*\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{R_{1,2}}(X)$ -open in U . The complement of nano $(1,2)^*$ -gs-closed set is called nano $(1,2)^*$ -gs-open set;
- (iv) nano $(1,2)^*$ - α g-closed set [8] if $N(1,2)^*\text{-}\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{R_{1,2}}(X)$ -open in U . The complement of

- nano (1,2)*- α g-closed set is called nano (1,2)*- α g-open set;
- (v) nano (1,2)*- \hat{g} -closed set [8] if $N_{\tau_{R_{1,2}}}$ - $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano (1,2)*-semi-open in U . The complement of nano (1,2)*- \hat{g} -closed set is called nano (1,2)*- \hat{g} -open set;
- (vi) nano (1,2)*- ψ -closed set [8] if $N(1,2)^*$ - $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano (1,2)*-sg-open in U . The complement of nano (1,2)*- ψ -closed set is called nano (1,2)*- ψ -open set;

Remark 2.9

- (i) Every $\tau_{R_{1,2}}$ -closed set is nano (1,2)*-semi-closed but not conversely [4].
- (ii) Every $N_{\tau_{R_{1,2}}}$ -closed set is nano (1,2)*- α -closed but not conversely [2].
- (iii) Every nano (1,2)*-semi-closed set is nano (1,2)*- ψ -closed but not conversely [8].
- (iv) Every nano (1,2)*-semi-closed set is nano (1,2)*-sg-closed but not conversely [8].
- (v) Every nano (1,2)*- \hat{g} -closed set is nano (1,2)*-g-closed but not conversely [8].
- (vi) Every nano (1,2)*-sg-closed set is nano (1,2)*-gs-closed but not conversely [8].
- (vii) Every nano (1,2)*-g-closed set is nano (1,2)*- α g-closed but not conversely [8].
- (viii) Every nano (1,2)*-g-closed set is nano (1,2)*-gs-closed but not conversely [8].

3. Nano (1,2)*- \tilde{g} -CLOSED SETS

We introduce the following definitions.

Definition 3.1

A subset A of a bitopological space U is called

- (i) nano (1,2)*- β -open set if $A \subseteq N_{\tau_{R_{1,2}}}$ - $\text{cl}(N_{\tau_{R_{1,2}}}$ - $\text{int}(N_{\tau_{R_{1,2}}}$ - $\text{cl}(A)))$.

The complements of nano (1,2)*- β -open set is called nano (1,2)*- β -closed set.

- (ii) nano (1,2)*- \tilde{g} -closed set if $N_{\tau_{R_{1,2}}}$ - $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano (1,2)*-sg-open in U .

- (iii) nano (1,2)*- \tilde{g} -closed if $N(1,2)^*$ - $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano (1,2)*- \hat{g} -open in U .
- (iv) nano (1,2)*- \tilde{g}_α -closed set if $N(1,2)^*$ - $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is nano (1,2)*-sg-open in U .
- (v) nano (1,2)*-gsp-closed set if $N(1,2)^*$ - $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $N_{\tau_{1,2}}$ -open in U .

The complements of the above mentioned closed sets are called their respective open sets.

Proposition 3.2

Every $\tau_{R_{1,2}}$ -closed set is nano (1,2)*- \tilde{g} -closed.

Proof

If A is a $\tau_{R_{1,2}}$ -closed subset of U and G is any nano (1,2)*-sg-open set containing A , then $G \supseteq A = N_{\tau_{R_{1,2}}}$ - $\text{cl}(A)$. Hence A is nano (1,2)*- \tilde{g} -closed in U .

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3 Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c, d\}\}$ and $X_1 = \{a, b\}$. Then $\tau_{R_1}(X_1) = \{\{\phi\}, U, \{a, b\}\}$, $U/R_2 = \{\{a\}, \{d\}, \{b, c\}\}$ and $X_2 = \{b, c\}$. Then $\tau_{R_2}(X_2) = \{\{\phi\}, U, \{b, c\}\}$. Then $\tau_{R_{1,2}}(X) = \{\{\phi\}, U, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Clearly,

the set $\{b, d\}$ is a nano (1,2)*- \tilde{g} -closed but it is not a $\tau_{R_{1,2}}$ -closed.

Proposition 3.4

Every nano (1,2)*- \tilde{g} -closed set is nano (1,2)*- \tilde{g}_α -closed.

Proof

If A is a nano (1,2)*- \tilde{g} -closed subset of U and G is any nano (1,2)*-sg-open set containing A , then

$G \supseteq N\tau_{R_{1,2}}\text{-cl}(A) \supseteq N(1,2)^*\text{-}\alpha\text{-cl}(A)$. Hence A is

nano $(1,2)^*\text{-}\ddot{g}_\alpha$ -closed in U.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c, d\}\}$ and $X_1 = \{a, b\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a, b\}\}$,

$U/R_2 = \{\{a\}, \{b, c\}, \{d\}\}$ and $X_2 = \{b, c\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{b, c\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a,$

$b\}, \{b, c\}, \{a, b, c\}\}$. Clearly, the set $\{a\}$ is an nano $(1,2)^*\text{-}\ddot{g}_\alpha$ -closed but not a nano $(1,2)^*\text{-}\ddot{g}$ -closed set in U.

Proposition 3.6

Every nano $(1,2)^*\text{-}\ddot{g}$ -closed set is nano $(1,2)^*\text{-}\psi$ -closed.

Proof

If A is a nano $(1,2)^*\text{-}\ddot{g}$ -closed subset of U and G is any nano $(1,2)^*\text{-sg}$ -open set containing A, then $G \supseteq N\tau_{R_{1,2}}\text{-cl}(A) \supseteq N(1,2)^*\text{-scl}(A)$. Hence A is nano $(1,2)^*\text{-}\psi$ -closed in U.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

In Example 3.5. Clearly, the set $\{c\}$ is a nano $(1,2)^*\text{-}\psi$ -closed but not a nano $(1,2)^*\text{-}\ddot{g}$ -closed set in U.

Proposition 3.8

Every nano $(1,2)^*\text{-}\ddot{g}$ -closed set is nano $(1,2)^*\text{-}\hat{g}$ -closed.

Proof

Suppose that $A \subseteq G$ and G is nano $(1,2)^*\text{-semi}$ -open in U. Since every nano $(1,2)^*\text{-semi}$ -open set is nano $(1,2)^*\text{-sg}$ -open and A is nano $(1,2)^*\text{-}\ddot{g}$ -closed, therefore $N\tau_{R_{1,2}}\text{-cl}(A) \subseteq G$. Hence A is nano $(1,2)^*\text{-}\hat{g}$ -closed in U.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

In Example 3.5. Clearly, the set $\{a, c\}$ is a nano $(1,2)^*\text{-}\hat{g}$ -closed but not a nano $(1,2)^*\text{-}\ddot{g}$ -closed set in U.

Proposition 3.10

Every nano $(1,2)^*\text{-}\alpha$ -closed set is nano $(1,2)^*\text{-}\ddot{g}_\alpha$ -closed.

Proof

If A is an nano $(1,2)^*\text{-}\alpha$ -closed subset of U and G is any nano $(1,2)^*\text{-sg}$ -open set containing A, we have $N(1,2)^*\text{-}\alpha\text{-cl}(A) = A \subseteq G$. Hence A is nano $(1,2)^*\text{-}\ddot{g}_\alpha$ -closed in U.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

In Example 3.5. Clearly, the set $\{a, c, d\}$ is an nano $(1,2)^*\text{-}\ddot{g}_\alpha$ -closed but not an nano $(1,2)^*\text{-}\alpha$ -closed set in U.

Remark 3.12

Nano $(1,2)^*\text{-}\hat{g}$ -closed set is different from nano $(1,2)^*\text{-}\tilde{g}$ -closed.

Example 3.13

- (i) In Example 3.5. Then $\{a\}$ is nano $(1,2)^*\text{-}\tilde{g}$ -closed set but not nano $(1,2)^*\text{-}\hat{g}$ -closed.
- (ii) In Example 3.5. Then $\{a, c\}$ is nano $(1,2)^*\text{-}\hat{g}$ -closed set but not nano $(1,2)^*\text{-}\tilde{g}$ -closed.

Proposition 3.14

Every nano $(1,2)^*\text{-}\ddot{g}$ -closed set is nano $(1,2)^*\text{-g}$ -closed.

Proof

If A is a nano $(1,2)^*\text{-}\ddot{g}$ -closed subset of U and G is any $N\tau_{1,2}$ -open set containing A, since every $\tau_{R_{1,2}}$ -open set is $(1,2)^*\text{-sg}$ -open, we have $G \supseteq N\tau_{R_{1,2}}\text{-cl}(A)$. Hence A is nano $(1,2)^*\text{-g}$ -closed in U.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

In Example 3.5. Clearly, the set {a, b, d} is a nano (1,2)*-g-closed but not a nano (1,2)*-g̃-closed set in U.

Proposition 3.16

Every nano (1,2)*-g̃-closed set is nano (1,2)*-αg-closed.

Proof

If A is a nano (1,2)*-g̃-closed subset of U and G is any τ_{R_{1,2}}-open set containing A, since every

τ_{R_{1,2}}-open set is nano (1,2)*-ĝ-open, we have

(1,2)*-α-Nτ_{R_{1,2}}-cl(A) ⊆ U. Hence A is nano

(1,2)*-αg-closed in U.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17

In Example 3.5, {b, d} is nano (1,2)*-αg-closed set but not nano (1,2)*-g̃-closed.

Proposition 3.18

Every nano (1,2)*-g̃-closed set is nano (1,2)*-αg-closed.

Proof

If A is a nano (1,2)*-g̃-closed subset of U and G is any τ_{R_{1,2}}-open set containing A, since every

τ_{R_{1,2}}-open set is nano (1,2)*-sg-open, we have G ⊇

Nτ_{R_{1,2}}-cl(A) ⊇ N(1,2)*-α cl(A). Hence A is nano

(1,2)*-αg-closed in U.

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19

In Example 3.5. Clearly, the set {c} is an nano (1,2)*-αg-closed but not a nano (1,2)*-g̃-closed set in U.

Proposition 3.20

Every nano (1,2)*-g̃-closed set is nano (1,2)*-gs-closed.

Proof

If A is a nano (1,2)*-g̃-closed subset of U and G is any τ_{R_{1,2}}-open set containing A, since every

τ_{R_{1,2}}-open set is nano (1,2)*-sg-open, we have G ⊇

Nτ_{R_{1,2}}-cl(A) ⊇ N(1,2)*-scl(A). Hence A is nano

(1,2)*-gs-closed in U.

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21

In Example 3.5. Clearly, the set {b, c, d} is a nano (1,2)*-gs-closed but not a nano (1,2)*-g̃-closed set in U.

Proposition 3.22

Every nano (1,2)*-g̃-closed set is nano (1,2)*-sg-closed.

Proof

If A is a nano (1,2)*-g̃-closed subset of U and G is any nano (1,2)*-semi-open set containing A, since every nano (1,2)*-semi-open set is nano

(1,2)*-sg-open, we have G ⊇ Nτ_{R_{1,2}}-cl(A) ⊇

N(1,2)*-scl(A). Hence A is nano (1,2)*-sg-closed in U.

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.23

In Example 3.5. Clearly, the set {c} is a nano (1,2)*-sg-closed but not a nano (1,2)*-g̃-closed set in U.

Proposition 3.24

Every nano (1,2)*-g̃_α-closed set is nano (1,2)*-g̃-closed.

Proof

If A is an nano (1,2)*-g̃_α-closed subset of U and G is any nano (1,2)*-ĝ-open set containing A,

since every nano $(1,2)^*$ - \hat{g} -open set is nano $(1,2)^*$ -sg-open, we have $N(1,2)^*-\alpha \text{ cl}(A) \subseteq G$. Hence A is nano $(1,2)^*$ - \tilde{g} -closed in U.

The converse of Proposition 3.24 need not be true as seen from the following example.

Example 3.25

In Example 3.5, {a, b, d} is nano $(1,2)^*$ - \tilde{g} -closed set but not nano $(1,2)^*$ - \tilde{g}_α -closed.

Proposition 3.26

Every nano $(1,2)^*$ - α -closed set is nano $(1,2)^*$ - \tilde{g} -closed.

Proof

If A is an nano $(1,2)^*$ - α -closed subset of U and G is any nano $(1,2)^*$ - \hat{g} -open set containing A, we have $N(1,2)^*-\alpha \text{ cl}(A) = A \subseteq G$. Hence A is nano $(1,2)^*$ - \tilde{g} -closed in U.

The converse of Proposition 3.26 need not be true as seen from the following example.

Example 3.27

In Example 3.5. Then {a, c, d} is nano $(1,2)^*$ - \tilde{g} -closed set but not nano $(1,2)^*$ - α -closed.

Proposition 3.28

Every nano $(1,2)^*$ - ψ -closed set is nano $(1,2)^*$ -sg-closed.

Proof

Suppose that $A \subseteq G$ and G is nano $(1,2)^*$ -semi-open in U. Since every nano $(1,2)^*$ -semi-open set is nano $(1,2)^*$ -sg-open and A is nano $(1,2)^*$ - ψ -closed, therefore $N(1,2)^*-\text{scl}(A) \subseteq G$. Hence A is nano $(1,2)^*$ -sg-closed in U.

The converse of Proposition 3.28 need not be true as seen from the following example.

Example 3.29

Let $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X_1 = \{a\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a\}\}$, $U/R_2 = \{\{a\}, \{b, c\}\}$ and $X_2 = \{b, c\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{b, c\}\}$.

Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a\}, \{b, c\}\}$. Clearly, the set {a, c} is a nano $(1,2)^*$ -sg-closed but not a nano $(1,2)^*$ - ψ -closed set in U.

Proposition 3.30

Every nano $(1,2)^*$ - \tilde{g} -closed set is nano $(1,2)^*$ -gsp-closed.

Proof

If A is a nano $(1,2)^*$ - \tilde{g} -closed subset of U and G is any $\tau_{R_{1,2}}$ -open set containing A, since every

$\tau_{R_{1,2}}$ -open set is nano $(1,2)^*$ -sg-open, we have $G \supseteq$

$N\tau_{R_{1,2}}-\text{cl}(A) \supseteq N(1,2)^*-\text{spcl}(A)$. Hence A is nano $(1,2)^*$ -gsp-closed in U.

The converse of Proposition 3.30 need not be true as seen from the following example.

Example 3.31

In Example 3.5. Clearly, the set {a, b} is a nano $(1,2)^*$ -gsp-closed but not a nano $(1,2)^*$ - \tilde{g} -closed set in U.

Proposition 3.32

Every nano $(1,2)^*$ - \hat{g} -closed set is nano $(1,2)^*$ -sg-closed.

Proof

If A is a nano $(1,2)^*$ - \hat{g} -closed subset of U and G is any nano $(1,2)^*$ -semi-open set containing A, then $G \supseteq N\tau_{R_{1,2}}-\text{cl}(A) \supseteq N(1,2)^*-\text{scl}(A)$. Hence A is nano $(1,2)^*$ -sg-closed in U.

The converse of Proposition 3.32 need not be true as seen from the following example.

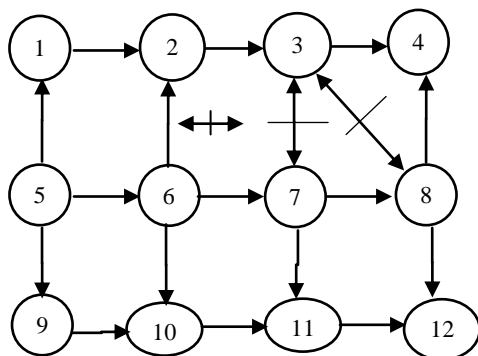
Example 3.33

In Example 3.5. Clearly, the set {a} is a nano $(1,2)^*$ -sg-closed but not a nano $(1,2)^*$ - \hat{g} -closed set in U.

closed sets. In future, I will discuss more applications of nano topological spaces.

Remark 3.34

From the above Propositions, Examples and Remark, we obtain the following diagram, where $A \rightarrow B$ (resp. $A \dashrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).



where

- (1) nano $(1,2)^*$ - α -closed
- (2) nano $(1,2)^*$ - \ddot{g}_α -closed
- (3) nano $(1,2)^*$ - \tilde{g} -closed
- (4) nano $(1,2)^*$ - α g-closed
- (5) $\tau_{R_{1,2}}$ -closed
- (6) nano $(1,2)^*$ - \ddot{g} -closed
- (7) nano $(1,2)^*$ - \hat{g} -closed
- (8) nano $(1,2)^*$ -g-closed
- (9) nano $(1,2)^*$ -semi-closed
- (10) nano $(1,2)^*$ - ψ -closed
- (11) nano $(1,2)^*$ -sg-closed
- (12) nano $(1,2)^*$ -gs-closed

Remark 3.35

The concepts of nano $(1,2)^*$ - \tilde{g} -closed sets and nano $(1,2)^*$ -g-closed sets are independent.

Example 3.36

- (i) In Example 3.5. Clearly $\{a, c\}$ is nano $(1,2)^*$ -g-closed set but it is not nano $(1,2)^*$ - \tilde{g} -closed set.
- (ii) In Example 3.5. Then $\{c\}$ is nano $(1,2)^*$ - \tilde{g} -closed set but it is not nano $(1,2)^*$ -g-closed set.

4. CONCLUSION

In this paper lies between the class of nano $(1,2)^*$ - \ddot{g}_α -closed sets and the class of nano $(1,2)^*$ - α g-

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